Modeling Clear-Air Turbulence with Vortices Using Parameter-Identification Techniques

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A vortex model of winds associated with clear-air turbulence is shown to be useful for characterizing actual airline encounters with severe turbulence. The model consists of an array of vortices with solid-body cores embedded in a potential flowfield. Parameters such as the size and strength of the vortices and their locations are identified using a modified Newton-Raphson algorithm. A manual identification startup scheme is used to minimize errors in the initial parameter estimates, and the identification algorithm is found to be robust in regard to the remaining errors. The analysis of a turbulence experience involving a commercial airliner demonstrates the success of the model and estimation procedure. The analysis finds vortices with core diameters of 1000 ft and tangential velocities of 87 ft/s in this encounter.

Introduction

CLEAR-air turbulence (CAT) is a little-understood phenomenon that occurs in the mid to upper troposphere and the lower stratosphere and that poses a safety hazard to aircraft. The most severe cases are characterized by sudden, violent disturbances with definite periodicity. This problem is being investigated by the NASA Ames Research Center in conjunction with the National Transportation Safety Board. Because of the small scale in both duration and distance over which CAT occurs, it has been a difficult phenomenon to study. However, in the recent past, wide-bodied commercial aircraft equipped with digital flight-data recorders have had a few encounters with CAT, and the data from these recorders, along with additional data from air-traffic control (ATC) radar records, have made it possible to obtain a detailed description of the winds associated with CAT encounters.

A model of the wind environment during CAT has been developed using these wind data and previous theoretical studies of CAT. These studies[1-6] predict that CAT is a result of the breakdown of stably stratified shear layers in the atmosphere, a condition known as Kelvin-Helmholtz instability. The shear layers roll up into a vortex array which forms in a Kelvin "cat's eyes" pattern (Fig. 1). The direction of rotation of the vortices depends on the way the shear layer breaks down, as discussed in Ref. 7.

To compare theory with data from actual encounters, parameter-identification techniques have been used to match the winds produced by a vortex-array model to the data. This has been done successfully with two actual encounters and is reported in a previous paper.[7] A similar technique had been used earlier to study an airplane's traling vortices.[8] The purpose of this paper is to describe the vortex model and the parameter-identification techniques used to match the model to the winds found in a CAT encounter.

First, a vortex model and its relation to an aircraft flight path is described. Next, the algorithm used to determine the model parameters is discussed briefly, followed by an explanation of the method used to obtain an initial guess of the parameters. Then, these methods are demonstrated with an example of an actual incident. The sensitivity of the estimation algorithm to errors in the initial guess is discussed, and finally there are some general comments on difficulties in the identification of vorties. The Appendix provides the implementation details of the parameter-identification procedure.

Vortex Model

The Kelvin cat's eyes pattern is modeled using two-dimensional flow theory and vortices with horizontal axes. The vortex model consists of a rotational (solid-body) core embedded in an irrotational flow. A plot of the velocity V induced in the flowfield as a function of the distance r from the vortex center is shown in Fig. 2, where r_0 is the radius of the solid-body core and V_0 the velocity tangent to the solid-body core. The velocity induced at a point is

$$V = \begin{cases} V_0 r_0 / r & \text{if} \quad r > r_0 \\ V_0 r / r_0 & \text{if} \quad r \le r_0 \end{cases} \tag{1}$$

Notice that there are two regions, one inside the core ($r \le r_0$) where the flow is rotational and one outside the core ($r > r_0$) where the flow is considered to be potential.

Each vortex axis is assumed to be horizontal and perpendicular to the wind vector. The flight path and vortex geometry used in calculating the wind perturbations during a turbulence encounter are shown in Fig. 3. The distances x and z are, respectively, the horizontal separation along the flight path and the vertical separation of the airplane from the vortex. The angle between the wind vector and the airplane's flight path is denoted by ψ. The distance of the airplane along a perpendicular to the line vortex is given by

$$r = (z^2 + x^2 \cos^2 \psi)^{1/2} \tag{2}$$

The components of the induced wind vector are

$$W_{xy} = Vz/r, \qquad W_z = V_x \cos(\psi)/r \tag{3}$$

where V is the velocity induced in the flowfield.

Some typical vortex-induced velocity perturbations that would be experienced by an aircraft passing a single vortex are illustrated in Fig. 4. The aircraft flight path is assumed to be perpendicular to the line vortex ($\psi = 0$). For passage directly through the core center ($z/r_0 = 0$), the vertical wind W_z has sharp peaks of $\pm V_0$ while the horizontal wind W_{xy} is zero. For other passage through the core ($z/r_0 < 1$), the peaks in W_z

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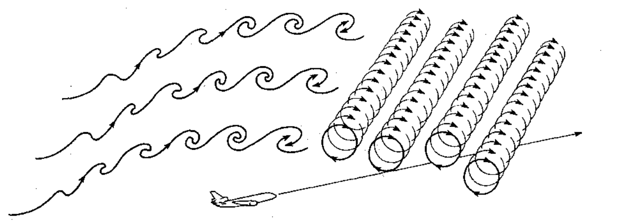

Fig. 1 Formation of vortices and subsequent arrangement in Kelvin's "cat's eyes" pattern.

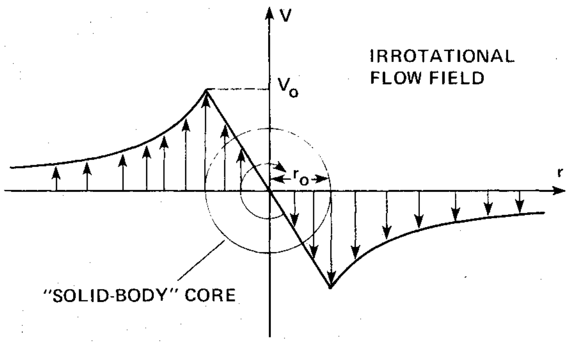

Fig. 2 Vortex model showing wind velocity as a function of distance from the vortex center.

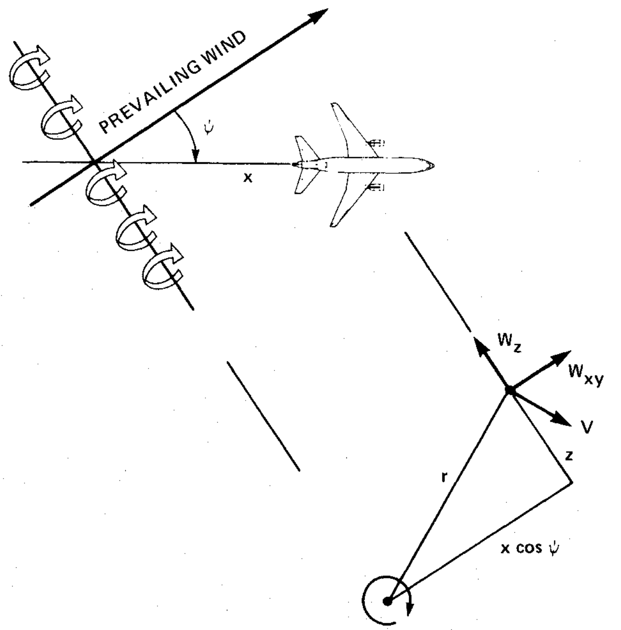

Fig. 3 Geometry used to represent the aircraft flight path with respect to a line vortex.

are attenuated but are still sharp and W_{xy} has a distinctive "plateau" feature which comes from the assumption of a solid-body core. For passage tangential to the core ($z/r_0 = 1$), W_z is even more attenuated and has smooth peaks and W_{xy} reaches its maximum value of V_0. For passage outside the core ($z/r_0 > 1$), both W_z and W_{xy} become smaller, until at large distances from the vortex there is little vortex-induced wind. These wind patterns, or "signatures," are very useful for obtaining rough estimates of the existence, positions, and characteristics of vortices from actual wind data.

The velocity perturbation induced by an array of vortices is modeled by superposition of the individual effects. The wind in both the horizontal and vertical directions at a point x_p along the flight path is modeled as

$$W_{xy} = b_{xy} + C_{xy}x_p \pm \sum_{i=1}^{n} \begin{cases} V_0 z_i r_0 / r_i^2 & \text{if } r_i > r_0 \\ V_0 z_i / r_0 & \text{if } r_i \le r_0 \end{cases}$$

$$W_z = b_z + C_z x_p - \sum_{i=1}^{n} \begin{cases} V_0 (x_p - x_i) \cos(\psi) r_0 / r_i^2 & \text{if } r_i > r_0 \\ V_0 (x_p - x_i) \cos(\psi) / r_0 & \text{if } r_i \le r_0 \end{cases} \tag{4}$$

where the constants b_{xy}, b_z, C_{xy}, and C_z model the non-vortex-related wind components (i.e., the bias and trend in the winds), and the subscript i refers to the ith vortex in an array of n vortices. Note that V_0 is a signed quantity where the sign depends on the sense of rotation of the vortex. If the airplane is flying to the right and the vortex is rotating clockwise, as pictured in Fig. 3, V_0 is positive. The wind perturbations induced by the vortex add to, or substract from, the prevailing wind, depending on the direction of the wind and the sense of rotation of V_0. The sign (+ or −) in computing W_{xy} is determined as follows: if the directions of the prevailing wind and the flight path are the same ($|\psi| < 90$ deg), the positive sign is used; if the directions of the prevailing wind and the flight path are opposite ($|\psi| > 90$ deg), the negative sign is used.

In summary, the parameters necessary to completely describe the situation are (x_i, z_i) the position of each vortex with respect to the flight path (i.e., the point where the vortex line passes through the vertical plane that includes the flight path); r_0, the radius of the vortex core (assumed to be the same for all vortices in the array); V_0, the tangential velocity of the vortex core (also assumed constant across the array); and the bias and trend constants for both the vertical and horizontal winds. These parameters are all determined by the estimation process.

Two additional parameters are not estimated by the algorithm, but are instead chosen initially by means of an inspection of the data. The angle ψ, and therefore the sign used in computing W_{xy}, is assumed constant over the duration of the encounter. That is, the vortex lines are assumed to be parallel. The value of ψ is chosen by inspecting the data record for the heading of the airplane and the direction of the wind during the encounter. The choice of the number of vortices, n, will become clear through the example.

Estimation Process

The parameters are to be determined such that a measure of the squared error between the actual and the model winds is minimized. The data for the horizontal wind velocity W_{xy} sometimes may not be as accurate as the data for the vertical wind velocity, and so the two errors may be given different weighting in the minimization process. Of the various minimization algorithms available, the modified Newton-Raphson (also called Gauss-Newton) was chosen because of its ability to handle the nonlinearity of the model. The implementation of the algorithm for this problem is described in the Appendix.

The Newton-Raphson algorithm performs very well when the estimate is near the value for minimum cost, but can have difficulty when the estimate is far from that value. This is because the algorithm uses the second gradient (or curvature matrix) of the cost function in determining the parameter estimates. If the surface of the cost function is not concave in the neighborhood of the initial estimate, the algorithm may lead away from the correct solution. Therefore, a separate startup scheme is often necessary to get close to the correct estimates before turning to the Newton-Raphson algorithm.[9] The startup scheme used in this case is simply the manual iden-

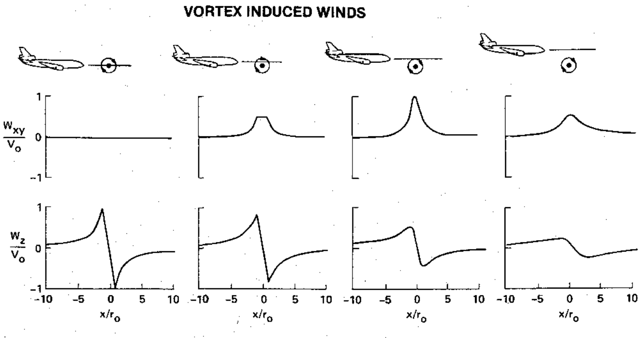

Fig. 4 Vortex-induced velocities encountered at various vertical separations for $\psi = 0$.

tification of vortices by comparing the wind patterns to model signatures, such as those shown in Fig. 4.

The number of vortices and their locations, and the strength V_0 and radius r_0 of the vortices can all be determined approximately from an inspection of the wind data. The process of estimating these parameters by inspection is best explained by an example. The performance of the minimization algorithm following the startup procedure will also be demonstrated with this example.

Example

Figure 5 shows plots of the vertical and horizontal winds during an encounter of a commercial airplane with CAT which occurred over Hannibal, Missouri, in July 1981. The winds were estimated using data from a digital flight-data recorder carried aboard the aircraft and from ATC radar records. The data and the estimated winds are all tabulated as functions of time. The x position for each data point is obtained by integrating the true airspeed and represents distance along the flight path from an arbitrary starting time. For the duration of this CAT encounter, the flight path was nearly straight and level at an altitude of 37,000 ft, with $\psi = 31$ deg.

The initial parameter estimates for the vortex model are obtained from the winds with the help of Fig. 4 and a good bit of judgment developmed over the course of this study. From Fig. 5, the horizontal wind has a bias of 150 knots; the vertical wind has no bias. So, $b_{xy} = 150$ knots and $b_z = 0$. Also, there is no significant trend in either the horizontal or vertical wind, so $C_{xy} = C_z = 0$.

The shape of the vertical wind velocity curve just to the left of $x = 0$ strongly indicates the presence of a vortex. The x coordinate for this vortex is chosen to be $x = -400$ ft because that is where $W_z = 0$. The horizontal wind at $x = -400$ ft shows only a small perturbation which indicates that the airplane passed quite close to the vortex core. The perturbations in W_z are approximately 65 ft/s, so if the airplane had gone through the core the perturbations would have been about 80 ft/s. This gives an initial value of $V_0 = 80$ ft/s. The distance between the minimum and maximum values of W_z around $x = -400$ ft is about 1200 ft. This is the distance that the plane flew inside the vortex, which leads to an estimate of $r_0 = 600$ ft. Since the plane passed inside the vortex and close to the core, let the z coordinate be $z = -150$ ft for this vortex. The negative sign comes from the perturbation in W_{xy} being negative (note that the perturbations in Fig. 4 are for a positive z). Another strong strong signature is found to the left of $x = 5000$ ft, with $W_z = 0$ at $x = 3300$ ft. The horizontal perturbation is also very strong, reaching a maximum of 35 knots, or about 60 ft/s. This indicates that the plane passed close to the edge of the vortex, and the sharpness of the vertical wind perturbation indicates that the plane flew through the vortex core; therefore, this suggests a value for the z coordinate of -500 ft.

In summary, the initial parameter estimates are two vortices of strength $V_0 = 80$ ft/s, size $r_0 = 600$ ft, and locations with a

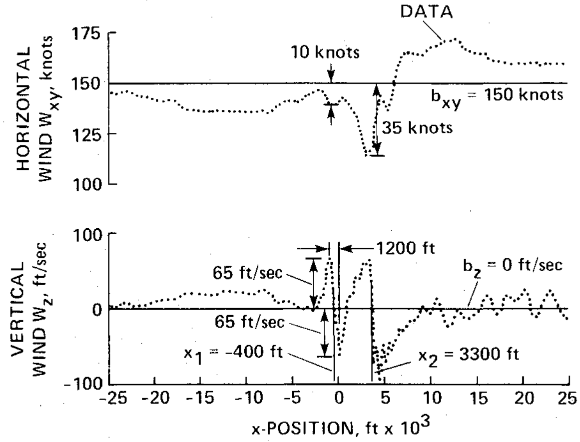

Fig. 5 Winds encountered during a CAT incident over Hannibal, MO, 1981, and extraction of some model parameters from data.

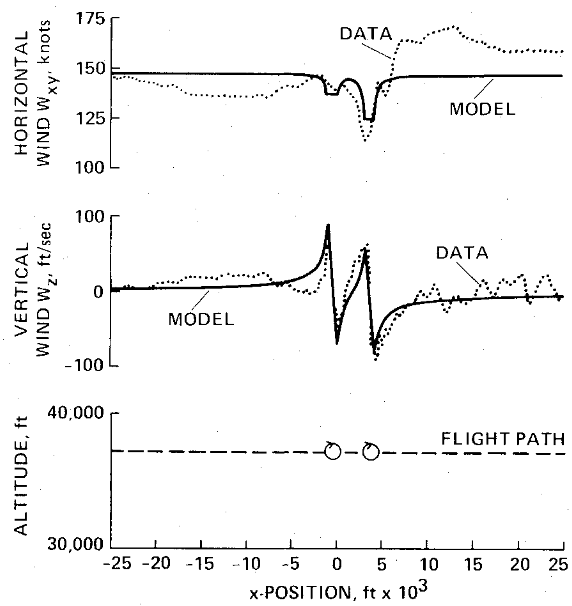

Fig. 6 Results of modeling winds using two vortices.

bias in the horizontal wind of 150 knots, as follows:

x, ft	z, ft
−400	−150
3300	−500

Using these initial estimates for the parameters, the modified Newton-Raphson algorithm minimizes the difference between the data and the model winds and determines a better set of parameters. The cost, as defined by Eq. (A3), with B an identity matrix, is 482 for the initial estimates. After the estimation process, the cost is 355 and the parameters are two vortices of strength $V_0 = 81.8$ ft/s, size $r_0 = 480$ ft, and locations with a bias in the horizontal wind of 147 knots, as follows:

x, ft	z, ft
−353	−96
3759	−221

Comparing these parameters with the initial estimates, it can be seen that the startup scheme performed quite well. However, Fig. 6, which is a plot of the model winds and the data, clearly shows that the match is not good, especially in the horizontal wind. The largest discrepancy in W_{xy} is in a region centered around $x = 12{,}000$ ft, so to improve the match

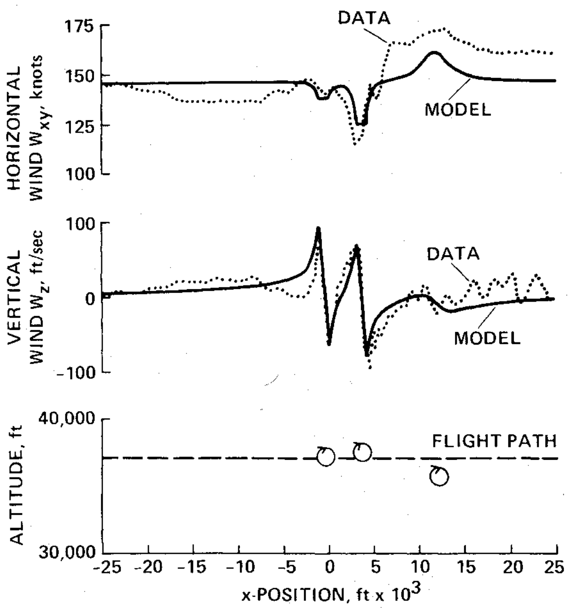

Fig. 7 Results of modeling winds using three vortices.

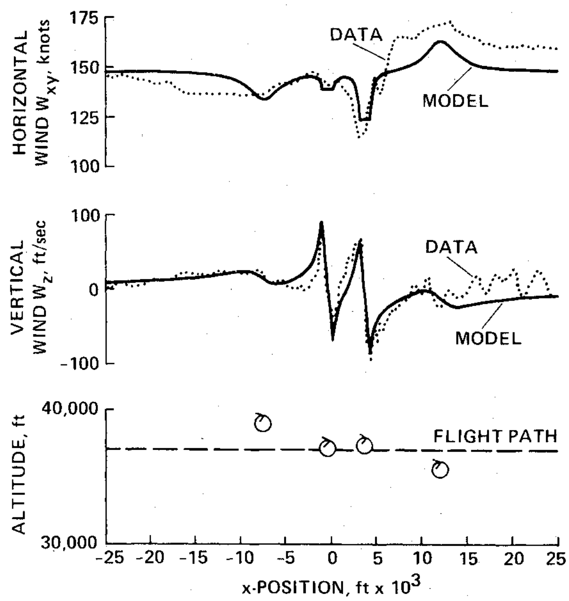

Fig. 8 Results of modeling winds using four vortices.

a third vortex is placed there. The perturbation in W_{xy} is positive and "rounded" so the vortex is placed well below the aircraft at $z = 2400$ ft. The resulting winds are shown in Fig. 7; the cost has been lowered to 303. Figure 8 shows the winds with four vortices, at a cost of 226. Finally, Fig. 9 shows five vortices, at a cost of 214. The final parameters for Fig. 9 are five vortices of strength $V_0 = 86.8$ ft/s, size $r_0 = 500.5$ ft, and locations with a horizontal wind bias of 149.8 knots, as follows.

x, ft	z, ft
−12,384	−3516
−6669	−1836
−343	−94
3761	−254
12,272	1738

The initial locations for the fourth and fifth vortices were also chosen by noting the perturbations in W_{xy}.

Further increases in the number of vortices ($n = 6,7$, etc.) do not result in decreases in the cost. In fact, the algorithm "pushes" the extra vortices away from the flight path, placing them at distances where their effects are negligible. These extra vortices, rather than resulting in a better fit, simply burden the estimation process. The algorithm converges quite rapidly. For the example with five vortices, Fig. 9, the parameters settled in six iterations, with small decreases in cost continuing until ten iterations.

Algorithm Sensitivity

As mentioned previously, the Newton-Raphson needs a startup scheme to obtain initial estimates that are sufficiently "close" to their actual values. To determine how significant this constraint was, the algorithm was used to estimate parameters from a set of data that was generated using the model itself. In this way, the parameter estimates obtained could be compared with the known parameters.

The parameters used to generate the data were $\psi = 0$ deg, $r_0 = 550$ ft, $V_0 = 85$ ft/s, $b_{xy} = 100$ knots, $b_z = 0$, and $n = 4$. These values were chosen because they are similar to those in many of the actual incidents studied.

With all other parameters correct, initial estimates for r_0 ranging from 100 to 1300 ft were used. The estimation process converged rapidly to the correct solution in all cases. Similarly, initial estimates of V_0 ranging from 30 to 250 ft/s resulted

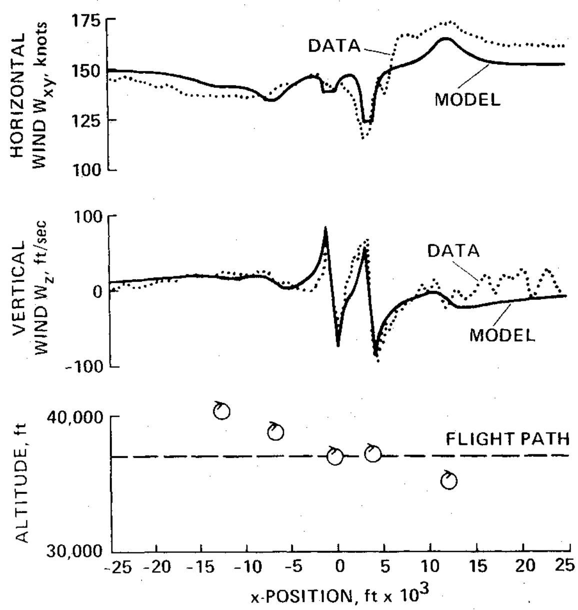

Fig. 9 Results of modeling winds using five vortices.

in the correct solution. The results of errors in the initial position of the vortices were similar, with the algorithm having little difficulty in determining the correct parameters. It seems, therefore, that the algorithm can tolerate quite large errors in the initial estimates, and that the "closeness" constraint does not pose a problem. The startup scheme described previously should certainly place the parameters within the range of convergence.

Discussion

The results of the example clearly did not result in a perfect match of data and model. This should not be surprising because the model [Eq. (4)] is meant to represent the sudden, violent, and periodic disturbances that characterize CAT, and not the small, random fluctuations that are part of the overall turbulence. It should be noted that the best match produced with five vortices, Fig. 9, shows a vortex pattern that is slanted and staggered, which agrees very well with theoretical predictions of vortex patterns developed from Kelvin-Helmholtz instability.

The inclusion of a strategy for setting the gain K in Eq. (A5) was prompted by concerns about difficulties the estimation algorithm may have because of a vortex model that consists of two separate regions. The two regions have different equations for the winds and gradients. It seems possible that a set of estimates p_k, which is in one region, could produce a Δp_{k+1} which would result in a set of new estimates p_{k+1} which is in the other region. In such a case, the algorithm does not guarantee a lower cost; in fact, the cost could be higher. The use of the gain strategy helps avoid such a situation, which fortunately happened infrequently in the cases studies. A more elegant solution would be to use a model that has just one set of equations and no sharp boundaries.

Conclusions

A model based on a vortex array in a potential flowfield has been developed to describe the wind environment associated with severe clear-air turbulence. The model generates winds with sudden and violent fluctuations that are characteristic of this phenomenon. Analysis of an actual aircraft turbulence encounter has demonstrated that this model accurately represents the winds. The development of the model and the parameter-estimation techniques will facilitate the analysis of other such encounters and further the knowledge of this phenomenon. The effects of severe turbulence on aircraft can also be studied by using the model to simulate turbulence encounters.

The modified Newton-Raphson algorithm, along with the startup scheme, has been shown to be effective in determining the model parameters from winds obtained from actual encounters. The startup scheme, which involves manual identification of the vortices by comparing the wind data to vortex signatures, is straghtforward and produces initial estimates of the parameters that are well within the range of convergence of the Newton-Raphson algorithm. Using the initial estimates, the algorithm converges in less than six iterations. In the turbulence encounter analyzed in the paper, the model matched the winds with five vortices. The vortices had diameters of 1000 ft and tangential velocities of 87 ft/s.

Appendix: Estimation Procedure

Let W_a be the actual wind and W_m the model wind

$$W_a = [W_{xy} \quad W_z]_a^T; \quad W_m = [W_{xy} \quad W_z]_m^T \tag{A1}$$

The error vector for the jth data point is then

$$e(j) = W_a(j) - W_m(j) \tag{A2}$$

A measure of the squared error over the N data points, the cost function, is

$$J = \frac{1}{N} \sum_{j=1}^{N} e^T(j) B e(j) \tag{A3}$$

where B is a weighting matrix. The parameters of the wind model are to be chosen such that J is minimized. The Newton-Raphson algorithm finds the minimum J iteratively by using both the gradient of the cost function with respect to the parameters and an estimate of its curvature. The iteration scheme is[10]

$$\Delta p_{k+1} = \left\{ \sum_{j=1}^{N} \left[\frac{\partial e}{\partial p}\right]_j^T B \left[\frac{\partial e}{\partial p}\right]_j \right\}^{-1} \left\{ \sum_{j=1}^{N} \left[\frac{\partial e}{\partial p}\right]_j^T B e(j) \right\} \tag{A4}$$

$$p_{k+1} = p_k - K\Delta p_{k+1} \tag{A5}$$

where p is the parameter vector

$$p = [(x_i, z_i), r_0, V_0, b_{xy}, b_z, C_{xy}, C_z]^T \quad (i = 1, \ldots, n) \tag{A6}$$

K is a gain, and $[\partial e/\partial p]$ is a sensitivity matrix, defined as

$$\left[\frac{\partial e}{\partial p}\right] \triangleq \begin{bmatrix} \dfrac{\partial e_{xy}}{\partial p_1} & \dfrac{\partial e_{xy}}{\partial p_2} & \cdots \\ \dfrac{\partial e_z}{\partial p_1} & \dfrac{\partial e_z}{\partial p_2} & \cdots \end{bmatrix} \tag{A7}$$

which is of dimension $2*(2n+6)$. Equations for the elements of the sensitivity matrix can be determined analytically by using Eqs. (A2), (A1), (A6), and (4), and these must be calculated for each iteration.

The gain K is necessary because the algorithm may overshoot and result in a higher cost. The gain is set as follows. The parameters are updated with $K=1$ and the cost is then determined for these new parameters. If this cost is less than the previous cost, the algorithm continues with the next update. If, however, the cost is not less than the previous cost, the value of K is halved. The parameters are again updated:

$$p_{k+1} = p_k - K\Delta p_{k+1}$$

The gain continues to be halved until either a lower cost is achieved or until the gain drops below some minimum value set by the user.

The update procedure continues until the cost is minimized (i.e., until the value of the gain falls below some threshold) or until a maximum number of iterations is reached.

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